Sensitivity and Uncertainty Analysis of the k_{eff} and β_{eff} for the ICSBEP and IRPhE Benchmarks

ANDES Workpackage N°: 3, Deliverable D3.3

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Prepared reports for Deliverable 3.3

- ***Ivan Kodeli, Cyrille De Saint Jean, Yannick Peneliau, ANALYSIS OF THE IRPhE AND ICSBEP EXPERIMENTS: C/E AND SENSITIVITY VECTORS***
- ***Ivan Kodeli, Cyrille De Saint Jean, Yannick Peneliau, SENSITIVITY PROFILES IN COMPUTER READABLE FORMAT***
- ***Ivan Kodeli,*** **SENSITIVITY AND UNCERTAINTY IN THE EFFECTIVE DELAYED NEUTRON FRACTION (β_{eff}), PHYSOR2012 presentation**

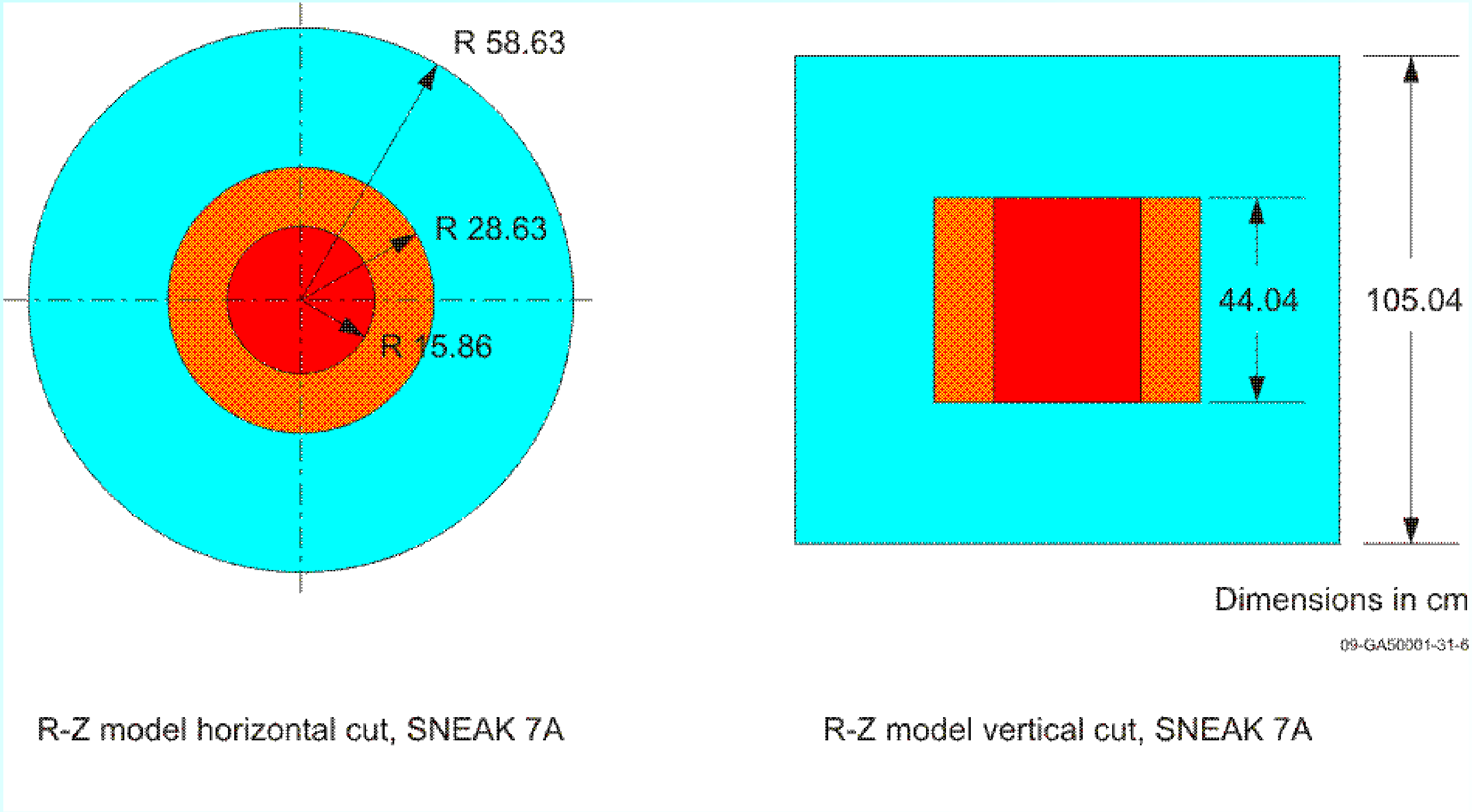
I. Analysis of benchmarks from ICSBEP and IRPhE

- **SNEAK-7A and -7B** fast critical experiments fuel with PuO_2 –UO_2 and reflected by metallic depleted uranium.
- **JEZABEL** is a bare sphere of plutonium metal
- **FLATTOP-Pu** benchmarks included a plutonium sphere reflected by natural uranium
- ZPPR10A experiment: sodium void effects have been measured in addition to the critical mass

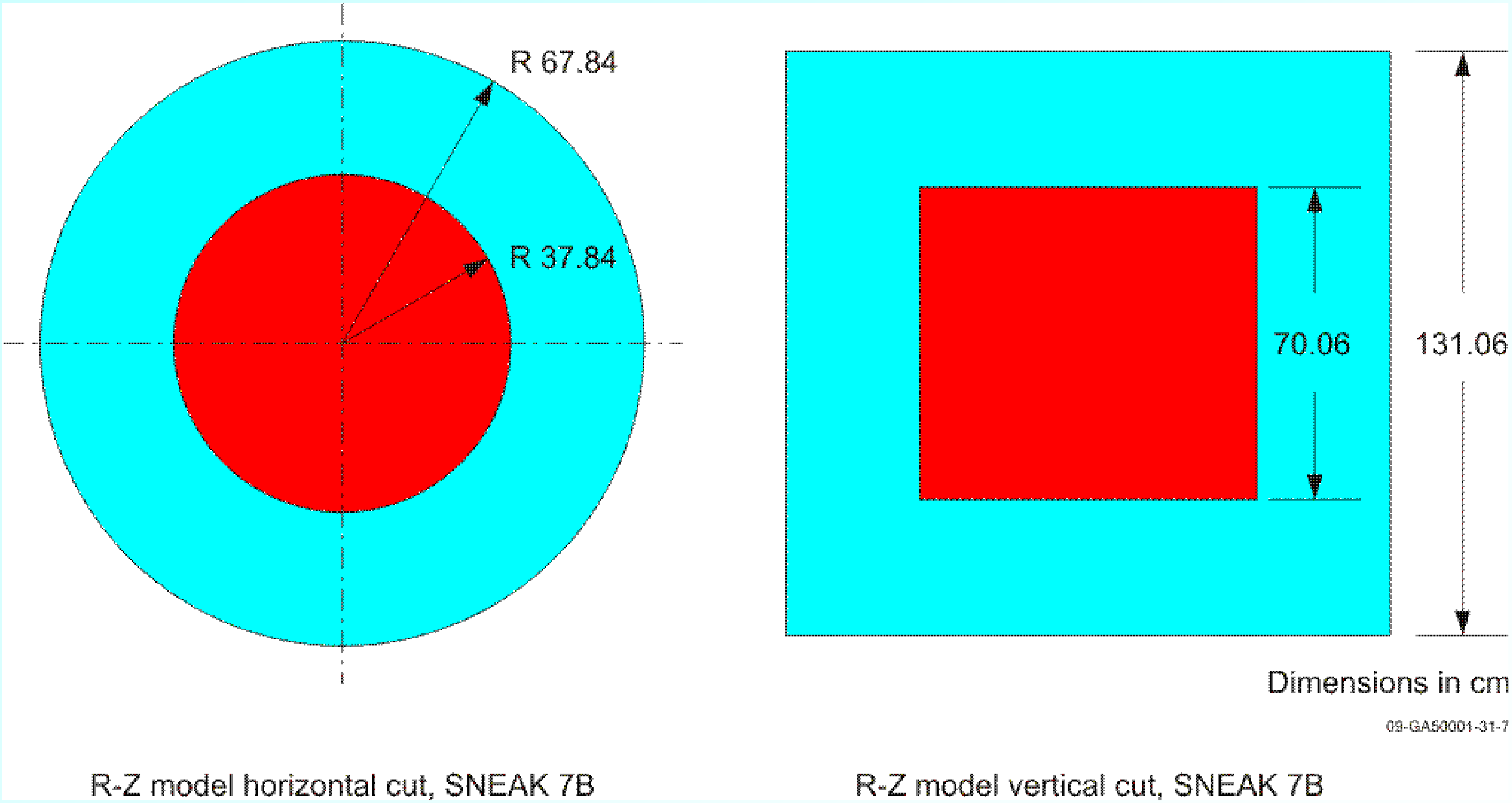
DESCRIPTION OF SNEAK-7 ASSEMBLIES

Reference: NEA/NSC/DOC(2006)1 March 2009 Edition – IRPhE Handbook (Evgeny Ivanov, IRSN, CNAM, IPN)

- Fuel element lattice pitch was 5.44 cm (square fuel elements tubes), and the cross-section of the platelets 5.077 x 5.077 cm^2.
- SNEAK-7A core unit cell consisted of one PuO_2-UO_2 pallet (26,6% PuO_2 and 73,4% UO_2) and one graphite platelet. Radial and axial blankets were loaded with depleted UO_2 plates.
- In SNEAK-7B the graphite platelets were replaced by $U_{nat}O_2$ platelet resulting in an average Pu-enrichment of ~13%.
- The effective core radii of SNEAK-7A & 7B were 28.63 cm and 37.84 cm, respectively.



SNEAK 7A R-Z Model



SNEAK 7B R-Z Model

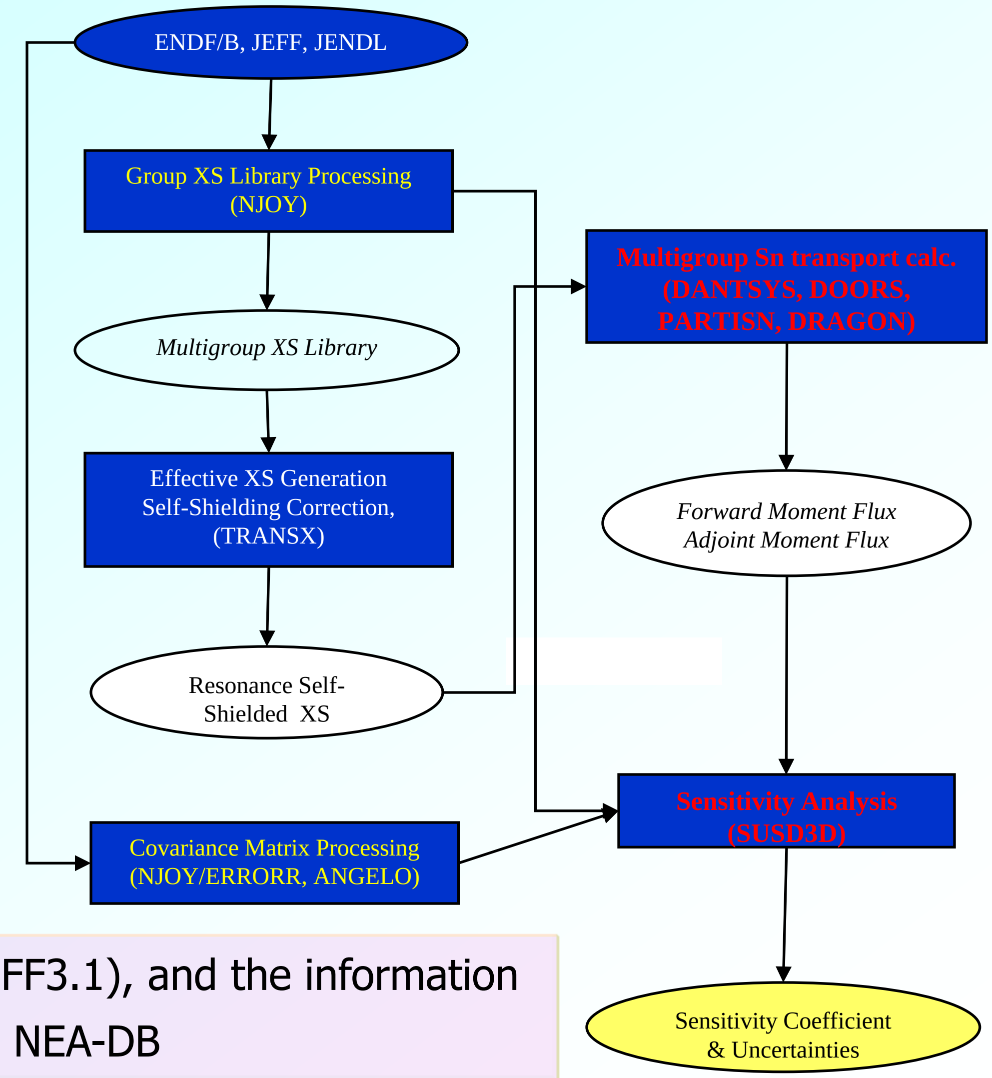
Model		SNEAK 7A		SNEAK 7B	
		k_{eff}	σk_{eff}	k_{eff}	σk_{eff}
1	Detailed 3D model with stretched platelets	1.0010 (a)	0.0029 (b)	1.0016 (a)	0.0035 (b)
2	3D simplified model with homogenized core regions of fuel elements	0.9998 (c)	0.0029	1.0001 (c)	0.0035
3	RZ two-dimensional model	1.0038 (c)	0.0029	1.0028 (c)	0.0035

(a) Measured k_{eff}.
(b) Combined experimental uncertainty
(c) Equal to experimental value + (calculated simplified model – calculated more detailed model)

	SNEAK 7A	SNEAK 7B
β_{eff} - ^{252}Cf source measurements	0.00395 5%	0.00429 5%
β_{eff} – noise analysis	0.00413 6%	0.00450 6%
β_{eff} – benchmark values	**0.00395 5%**	**0.00429 6%**

JSI Computational Methods and Procedures

- DANTSYS
  - -SN transport code (1D, 2D, 3D)
- TRANSX
  - - Processes MATXS format library for SN transport code
  - - Self-shielding factor method
- SUSD3D
  - - Multi-dimensional, SN based XS sensitivity and uncertainty
  - - First-order perturbation theory



- All computer codes, XS library (KAFAX, JEFF3.1), and the information on benchmarks publicly available through NEA-DB

CEA Computational Methods and Procedures

- **Neutronic and sensitivity codes : ERANOS and PARIS**
- **Cross sections: Ecco+Sn2d with JEFF 3.1.1 and ENDF/B-VII (33 energy groups)**
- **Sensitivities to cross sections, total nu-bar and fission spectra**

SNEAK-7A & -7B, JEZEBEL, FLATTOP

XS from ENDF/B-VII processed by NJOY (80/33 n groups)

SNEAK-7A: TWODANT 2D, r=58.63 cm, z=52.52 cm, 49x52I, P3, S8, k_{eff}=1.00733292 (dir), 1.00732422 (adj)

SNEAK-7B: TWODANT 2D, r=67.84 cm, z=65.53 cm, 68x65 I, P3, S8, k_{eff}=1.00213408 (dir), 1.00213277 (adj)

JEZEBEL-Pu239: ONEDANT 1D, r=6.3849 cm, 60I, P_5, S_{48}

k_{eff}=0.998396 (dir), 0.998397 (adj)

FLATTOP: ONEDANT 1D, r_1=4.5332 cm, r_2=24.1420cm, 725I, P_5, S_{48}

k_{eff}=1.00143 (dir), 1.00141 (adj)

TOPSY: ONEDANT 1D, r_1=6.1156 cm, r_2= 24.1242 cm, 237I, P_5, S_{48}

ZPR 6-7: TWODANT 2D R-Z

NEA-1650 ZZ-KAFAX-F22 (80 n groups JEFF 2.2) k_{eff}=0.985169 (P_1S_4)

NEA-1815: ZZ-KAFAX-E70 150 n groups ENDF/B-VII.0 (^{238}U data recalculated since k_{eff}~1.08 was obtained with original ^{238}U)

k_{eff}=1.0009163 (P_1S_4, direct) k_{eff}=1.0009171 (P_1S_4, adjoint)

k_{eff}=1.0006591 (P_3S_4, direct)

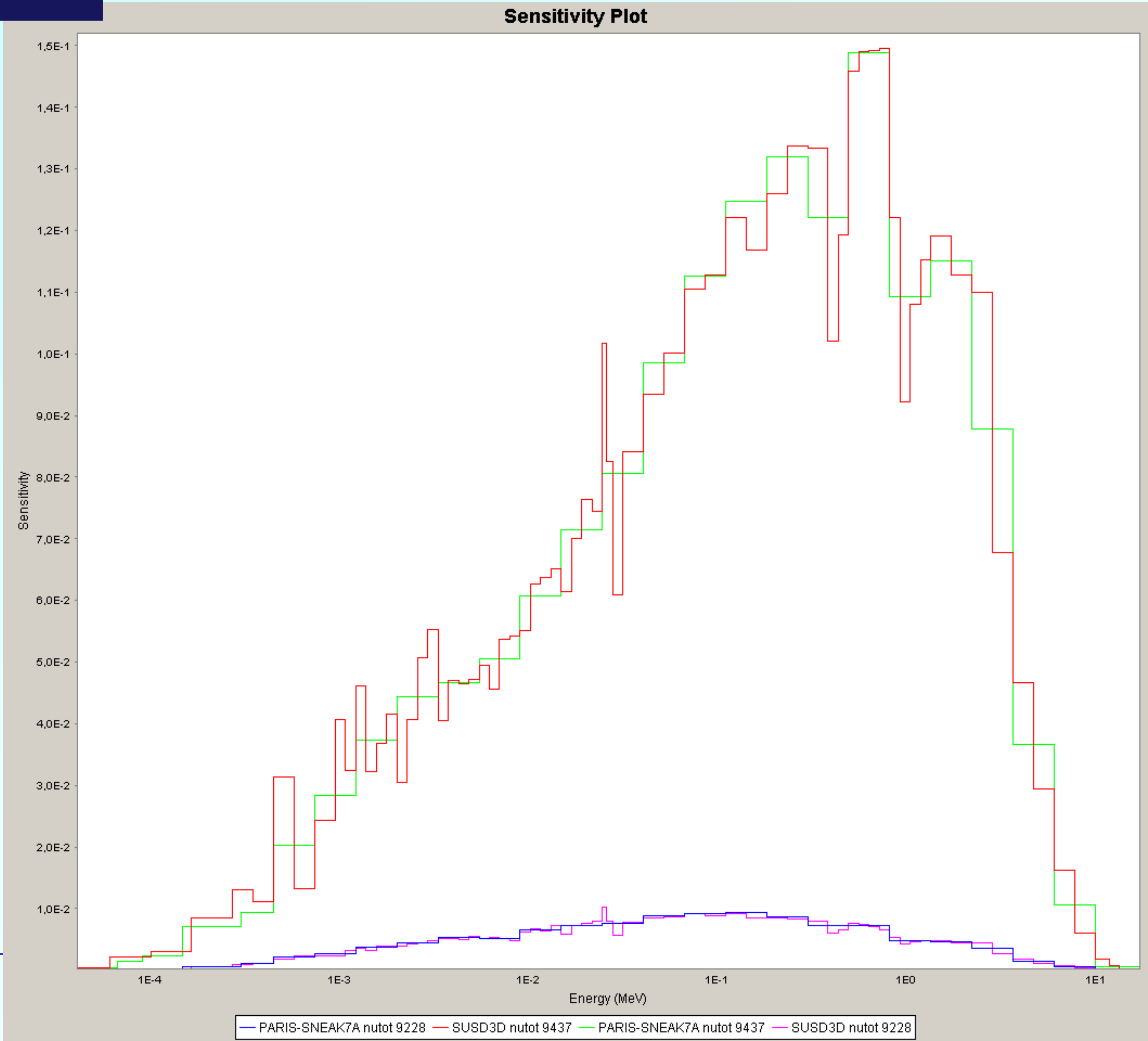
SNEAK-7A

SNEAK-7A / SUSD3D ENDF/BVII	Sensitivity (%/%)					
	Elastic	Inelastic	(n,2n)	(n,f)	(n,γ)	ν_{tot}
^{235}U	$8.74 \cdot 10^{-4}$	$-2.52 \cdot 10^{-4}$	$1.42 \cdot 10^{-5}$	$3.65 \cdot 10^{-2}$	$-5.16 \cdot 10^{-3}$	$5.56 \cdot 10^{-2}$
^{238}U	$9.95 \cdot 10^{-2}$	$-1.76 \cdot 10^{-2}$	$8.80 \cdot 10^{-4}$	$8.66 \cdot 10^{-2}$	$-1.49 \cdot 10^{-1}$	$1.36 \cdot 10^{-1}$
^{239}Pu	$6.43 \cdot 10^{-3}$	$-1.38 \cdot 10^{-3}$	$6.03 \cdot 10^{-5}$	$5.39 \cdot 10^{-1}$	$-5.91 \cdot 10^{-2}$	$7.79 \cdot 10^{-1}$
^{240}Pu	$6.74 \cdot 10^{-4}$	$-1.74 \cdot 10^{-4}$	$3.26 \cdot 10^{-6}$	$1.39 \cdot 10^{-2}$	$-5.47 \cdot 10^{-4}$	$2.02 \cdot 10^{-2}$
^{241}Pu	$4.54 \cdot 10^{-5}$	$-1.81 \cdot 10^{-5}$	$3.21 \cdot 10^{-6}$	$5.96 \cdot 10^{-3}$	$-4.46 \cdot 10^{-4}$	$8.62 \cdot 10^{-3}$
^{242}Pu	$3.89 \cdot 10^{-6}$	$-1.09 \cdot 10^{-6}$	$4.47 \cdot 10^{-8}$	$4.83 \cdot 10^{-5}$	$-2.57 \cdot 10^{-5}$	$7.05 \cdot 10^{-5}$

SNEAK-7A / PARIS ENDF/BVII	**Sensitivity (%/%)**					
	Elastic	**Inelastic**	**(n,xn)**	**(n,f)**	(n,γ)	ν_{prompt}
^{235}U	$8.82 \cdot 10^{-4}$	$-2.16 \cdot 10^{-4}$	$1.33 \cdot 10^{-5}$	$3.68 \cdot 10^{-2}$	$-5.20 \cdot 10^{-3}$	$5.59 \cdot 10^{-2}$
^{238}U	$9.92 \cdot 10^{-2}$	$-1.47 \cdot 10^{-2}$	$9.52 \cdot 10^{-4}$	$8.64 \cdot 10^{-2}$	$-1.68 \cdot 10^{-1}$	$1.35 \cdot 10^{-1}$
^{239}Pu	$6.39 \cdot 10^{-3}$	$-1.10 \cdot 10^{-3}$	$5.76 \cdot 10^{-5}$	$5.39 \cdot 10^{-1}$	$-6.03 \cdot 10^{-2}$	$7.80 \cdot 10^{-1}$
^{240}Pu	$6.78 \cdot 10^{-4}$	$-1.49 \cdot 10^{-4}$	$3.31 \cdot 10^{-6}$	$1.37 \cdot 10^{-2}$	$-5.62 \cdot 10^{-3}$	$1.99 \cdot 10^{-2}$
^{241}Pu	$4.52 \cdot 10^{-5}$	$-1.46 \cdot 10^{-5}$	$2.92 \cdot 10^{-6}$	$5.97 \cdot 10^{-3}$	$-4.45 \cdot 10^{-4}$	$8.64 \cdot 10^{-3}$
^{242}Pu	$3.38 \cdot 10^{-6}$	$-1.16 \cdot 10^{-6}$	$4.22 \cdot 10^{-8}$	$4.82 \cdot 10^{-5}$	$-2.31 \cdot 10^{-5}$	$6.98 \cdot 10^{-5}$

SNEAK-7A

Sensitivity plot



SNEAK-7B

SNEAK-7B /SUSD3D ENDF/BVII 80-groups	Sensitivity (%/%)					
	Elastic	Inelastic	(n,2n)	(n,f)	(n,γ)	ν_{tot}
^{235}U	$6.80\cdot10^{-4}$	$-1.01\cdot10^{-3}$	$1.31\cdot10^{-5}$	$6.25\cdot10^{-2}$	$-7.31\cdot10^{-3}$	$9.13\cdot10^{-2}$
^{238}U	$7.69\cdot10^{-2}$	$-6.71\cdot10^{-2}$	$7.36\cdot10^{-4}$	$1.14\cdot10^{-1}$	$-2.43\cdot10^{-1}$	$1.86\cdot10^{-1}$
^{239}Pu	$3.67\cdot10^{-3}$	$-3.63\cdot10^{-3}$	$2.46\cdot10^{-5}$	$5.08\cdot10^{-1}$	$-3.85\cdot10^{-2}$	$6.99\cdot10^{-1}$
^{240}Pu	$3.89\cdot10^{-4}$	$-3.58\cdot10^{-4}$	$1.09\cdot10^{-6}$	$1.12\cdot10^{-2}$	$-3.84\cdot10^{-3}$	$1.61\cdot10^{-2}$
^{241}Pu	$2.55\cdot10^{-5}$	$-4.49\cdot10^{-5}$	$1.40\cdot10^{-6}$	$5.59\cdot10^{-3}$	$-3.17\cdot10^{-4}$	$7.63\cdot10^{-3}$
^{242}Pu	$1.97\cdot10^{-6}$	$-1.80\cdot10^{-6}$	$1.50\cdot10^{-8}$	$3.37\cdot10^{-5}$	$-1.60\cdot10^{-5}$	$4.91\cdot10^{-5}$

SNEAK-7B /PARIS ENDF/BVII	Sensitivity (%/%)					
	Elastic	Inelastic	(n,xn)	(n,f)	(n,γ)	ν_{prompt}
^{235}U	$6.83\cdot10^{-4}$	$-9.48\cdot10^{-4}$	$1.27\cdot10^{-5}$	$6.25\cdot10^{-2}$	$-7.30\cdot10^{-3}$	$9.14\cdot10^{-2}$
^{238}U	$7.39\cdot10^{-2}$	$-6.32\cdot10^{-2}$	$8.29\cdot10^{-4}$	$1.13\cdot10^{-1}$	$-2.27\cdot10^{-1}$	$1.83\cdot10^{-1}$
^{239}Pu	$3.66\cdot10^{-3}$	$-3.41\cdot10^{-3}$	$2.49\cdot10^{-5}$	$5.08\cdot10^{-1}$	$-3.99\cdot10^{-2}$	$7.02\cdot10^{-1}$
^{240}Pu	$3.94\cdot10^{-4}$	$-3.38\cdot10^{-4}$	$1.39\cdot10^{-6}$	$1.11\cdot10^{-2}$	$-3.98\cdot10^{-3}$	$1.60\cdot10^{-2}$
^{241}Pu	$2.53\cdot10^{-5}$	$-4.19\cdot10^{-5}$	$1.30\cdot10^{-6}$	$5.60\cdot10^{-3}$	$-3.16\cdot10^{-4}$	$7.65\cdot10^{-3}$
^{242}Pu	$1.76\cdot10^{-6}$	$-2.02\cdot10^{-6}$	$1.60\cdot10^{-8}$	$3.39\cdot10^{-5}$	$-1.47\cdot10^{-5}$	$4.90\cdot10^{-5}$

JEZEBEL

JEZABEL /SUSD3D ENDF/BVII	Sensitivity (%/%)				
	Elastic	Inel.	(n,γ)	(n,f)	ν_{tot}
^{239}Pu	$6.44 \cdot 10^{-2}$	$4.53 \cdot 10^{-2}$	$-7.59 \cdot 10^{-3}$	$7.29 \cdot 10^{-1}$	$9.66 \cdot 10^{-1}$
^{240}Pu	$3.55 \cdot 10^{-3}$	$1.91 \cdot 10^{-3}$	$-6.15 \cdot 10^{-4}$	$2.35 \cdot 10^{-2}$	$3.13 \cdot 10^{-2}$
^{241}Pu	$1.69 \cdot 10^{-4}$	$1.56 \cdot 10^{-4}$	$-4.54 \cdot 10^{-5}$	$2.10 \cdot 10^{-3}$	$2.79 \cdot 10^{-3}$

JEZABEL /PARIS JEFF3.1.1	Sensitivity (%/%)				
	Elastic	Inelastic	Capture	(n,f)	ν_{prompt}
^{239}Pu	$6.77 \cdot 10^{-2}$		$-9.22 \cdot 10^{-3}$	$7.27 \cdot 10^{-1}$	$9.65 \cdot 10^{-1}$
^{240}Pu	$3.54 \cdot 10^{-3}$	$2.01 \cdot 10^{-3}$	$-6.42 \cdot 10^{-4}$	$2.40 \cdot 10^{-2}$	$3.19 \cdot 10^{-2}$
^{241}Pu	$2.51 \cdot 10^{-4}$	$6.47 \cdot 10^{-5}$	$-5.89 \cdot 10^{-5}$	$2.19 \cdot 10^{-3}$	$2.90 \cdot 10^{-3}$

FLATTOP

FLATTOP /SUSD3D ENDF/BVII	Sensitivity (%/%)				
	Elastic	Inelastic	(n,f)	(n,γ)	ν_{tot}
^{235}U	$8.15\cdot10^{-4}$	$3.48\cdot10^{-4}$	$7.58\cdot10^{-3}$	$-5.44\cdot10^{-4}$	$1.05\cdot10^{-2}$
^{238}U	$1.39\cdot10^{-1}$	$6.53\cdot10^{-2}$	$5.79\cdot10^{-2}$	$-3.98\cdot10^{-2}$	$7.90\cdot10^{-2}$
^{239}Pu	$2.21\cdot10^{-2}$	$1.24\cdot10^{-2}$	$6.33\cdot10^{-1}$	$-1.29\cdot10^{-2}$	$8.81\cdot10^{-1}$
^{240}Pu	$1.27\cdot10^{-3}$	$5.92\cdot10^{-4}$	$1.91\cdot10^{-2}$	$-9.91\cdot10^{-4}$	$2.66\cdot10^{-2}$
^{241}Pu	$6.00\cdot10^{-5}$	$5.07\cdot10^{-5}$	$1.94\cdot10^{-3}$	$-6.31\cdot10^{-5}$	$2.71\cdot10^{-3}$

FLATTOP /PARIS JEFF3.1.1	Sensitivity (%/%)				
	Elastic	Inelastic	Capture	(n,f)	ν_{prompt}
^{235}U	$7.87\cdot10^{-4}$	$4.01\cdot10^{-4}$	$-5.69\cdot10^{-4}$	$7.94\cdot10^{-3}$	$1.10\cdot10^{-2}$
^{238}U	$1.38\cdot10^{-1}$	$7.63\cdot10^{-2}$	$-4.22\cdot10^{-2}$	$6.13\cdot10^{-2}$	$8.10\cdot10^{-2}$
^{239}Pu	$2.20\cdot10^{-2}$		$-1.46\cdot10^{-2}$	$6.31\cdot10^{-1}$	$8.79\cdot10^{-1}$
^{240}Pu	$1.33\cdot10^{-3}$	$7.95\cdot10^{-4}$	$-1.03\cdot10^{-3}$	$1.93\cdot10^{-2}$	$2.66\cdot10^{-2}$
^{241}Pu	$9.43\cdot10^{-5}$	$2.16\cdot10^{-5}$	$-8.15\cdot10^{-5}$	$2.01\cdot10^{-3}$	$2.80\cdot10^{-3}$

CONCLUSIONS I (SU Intercomparison)

- **Different approached developed at JSI and CEA, using different computer codes (SUSD3D, ERANOS, PARIS), cross-section libraries, energy group structures and modelisation of the geometry were applied to a series of high quality benchmark experiments, including SNEAK-7A & -7B, JEZEBEL and FLATTOP fast reactor systems. This study confirms good agreement between the sensitivities, both of integral values and sensitivity profiles, calculated using the two approaches and contributes in this way to their validation.**

II. Sensitivity / Uncertainty of the Effective Delayed Neutron Fraction

SUMMARY RECORD of UAM-4

- Evaluate uncertainties in delayed neutron yield and its distribution in six delayed groups.
- Add SNEAK (fast core problem) as an optional test case to the test problems for Exercise I-3 since it has a unique set of experimental data for βeff uncertainties and can be used as an example on how to calculate uncertainty in βeff. The two high-quality reactor physics benchmark experiments, SNEAK-7A & 7B (Karlsruhe Fast Critical Facility) are part of the International Reactor Physics Benchmark Experiments (IRPhE) database. The objectives of adding SNEAK test problem to Exercise I-3 are as follow:
  - The PWR, BWR & VVER cases are very similar in spectrum, a fast reactor case based on a well evaluated experiment would broaden the verification of methods for Phase I of UAM and is a link to reactors of a next generation;
  - Fast reactor benchmarks are proposed to test and compare state-of-the-art cross section sensitivity and uncertainty codes;
  - Benchmark provides an unique set of experimental data on delayed neutrons effective fraction; it could be useful in analysis of components of a βeff uncertainty;
  - The analysis of the experimental set could bring a better comprehension on validity of covariance matrices to be applied.

Effective delayed neutron fraction β-eff

$$\beta_{eff} = \frac{P_{d,eff}}{P_{eff}}$$

$$P_{d,eff} = \int \Phi^{+}(\vec{r}, E', \Omega')\chi_d(E')dE'd\Omega' \int \nu_d(E)\Sigma_f(\vec{r}, E)\Phi(\vec{r}, E, \Omega)dEd\Omega d\vec{r}$$

$$P_{eff} = \int \Phi^{+}(\vec{r}, E', \Omega')\chi(E')dE'd\Omega' \int \nu(E)\Sigma_f(\vec{r}, E)\Phi(\vec{r}, E, \Omega)dEd\Omega d\vec{r}$$

where:

- Φ, Φ^{+} = direct, adjoint angular fluxes,
- Σ_f = fission cross section,
- χ_d, χ = fission spectra (delayed & total),
- ν_d, ν = delayed & total nu-bar.

Effective delayed neutron fraction β-eff

DETERMINISTIC CODES: easy

β_{eff} can be obtained e.g. using SUSD3D as a ration of two sensitivity terms:

$$\beta_{eff} = \frac{\int S_{\nu_d}(E)dE}{\int \left(S_{\nu_p}(E) + S_{\nu_d}(E) \right) dE}$$

$$S_{\nu_p}(E) = \frac{1}{R}\int d\vec{r}\int d\vec{\Omega}\left[\int d\vec{\Omega}'\int dE' \cdot \Phi(\vec{r},\vec{\Omega},E)\cdot\Phi^*(\vec{r},\vec{\Omega}',E')\chi_p(E')\nu_p(E)\sigma_f^i(E)N^i(\vec{r})\right]$$

$$S_{\nu_d}(E) = \frac{1}{R}\int d\vec{r}\int d\vec{\Omega}\left[\int d\vec{\Omega}'\int dE' \cdot \Phi(\vec{r},\vec{\Omega},E)\cdot\Phi^*(\vec{r},\vec{\Omega}',E')\chi_d(E')\nu_d(E)\sigma_f^i(E)N^i(\vec{r})\right]$$

Monte Carlo: M. M. Bretscher

$$\beta_{eff} = 1 - \frac{k_p}{k}$$

Calulated & measured **β-eff**

β-eff (pcm)	SUSD3D	MCNP	Measured
SNEAK-7A	373	369	395 5%
SNEAK-7B	420	415	429 6%
FLATTOP-Pu	277	284	360 9%
JEZEBEL	184	186	194 10%

Sensitivity & uncertainty in β-eff

Hammer & Tuttle (1979): $\Delta\beta_{eff}/\beta_{eff} \approx 5\%$ (power reactor)
(Φ, Φ^+ ~2%, $\nu\sigma_f$~1.5%, χ_d~0.5%)
D'Angelo et al. (1987): $\Delta\beta_{eff}/\beta_{eff} \approx 5\%$ (fast reactors)
(ν_d~2%, χ_d~0.5%)
Zukeran et al. (1999): $\Delta\beta_{eff}/\beta_{eff} \approx$4-5% (generalised pert. method)
(ν_d~2.5%, σ_f~1.6%, χ_d~0.5%)

- P. Hammer, Requirements of delayed neutron data for the design, operation, dynamics and safety of fast breeder and thermal power reactors, *Proc. Consultants Meeting on Delayed Neutron Properties, Vienna, 26-30 March 1979*, INDC (NDS)-107/G+Special
- *D'Angelo A., Vuillemin B. and Cabrillat J.C.: NEACRP-A-766, (1987).*
- *A. Zukeran, H. Hanaki, S. Sawada, T. Suzuki, "Uncertainty Evaluation of Effecti ve Delayed Neutron Fraction β_{eff} of Typical Proto-type Fast Reactor," J. Nucl. Sci. and Technology, **Vol. 36**, No.1, p. 61-80 (Jan. 1999).*

Sensitivity & uncertainty in β-eff

Monte Carlo: M. M. Bretscher

$$\beta_{eff} = 1 - \frac{k_p}{k}$$

Sensitivity of β-eff can be obtained as a (properly weighted) difference between two standard sensitivity terms:

$$\frac{\sigma}{\beta_{eff}}\frac{\partial \beta_{eff}}{\partial \sigma} = \frac{\sigma}{\beta_{eff}}\left(-\frac{\partial k_p}{k\partial\sigma} + \frac{k_p \partial k}{k^2 \partial\sigma}\right) = \frac{k_p}{k-k_p}\left(-\frac{\sigma}{k_p}\frac{\partial k_p}{\partial\sigma} + \frac{\sigma}{k}\frac{\partial k}{\partial\sigma}\right) = \frac{k_p}{k-k_p}\left(S_k - S_{kp}\right)$$

or alternatively:

$$\frac{\sigma}{\beta_{eff}}\frac{\partial \beta_{eff}}{\partial \sigma} = \frac{k_p}{k\beta_{eff}}\left(S_k - S_{kp}\right)$$

This method was first proposed in:

- I. Kodeli, Sensitivity and uncertainty in the effective delayed neutron fraction β-eff (method and SNEAK-7A example), proc. UAM-5 Workshop, Stockholm, Apr. 13-15, 2011, NEA-1769/04 package, OECD-NEA Data Bank.

- I. Kodeli, Sensitivity And Uncertainty in the Effective Delayed Neutron Fraction (β_{eff}), *PHYSOR 2012* – Knoxville, Tennessee, USA, April 15-20, 2012, on CD-ROM (2012).

In January 2012 a similar method was independently presented in:

- G. Chiba et al., "JENDL-4.0 Benchmarking for Effective Delayed Neutron Fraction of Fast Neutron Systems, " *J. Nucl. Sci. Technology*, Vol. 48, No. 12, p.1471-1477 (2011), available online 5. Jan. 2012.

SNEAK-7A: Sensitivity to β-eff

	Sensitivity (%/%)					
MAT	elastic	inelastic	(n,f)	(n,γ)	ν_{del}	ν_{pmt}
U-235	$-4.05 \cdot 10^{-4}$	$-1.81 \cdot 10^{-3}$	**$4.10 \cdot 10^{-2}$**	$5.18 \cdot 10^{-4}$	**$7.54 \cdot 10^{-2}$**	**$-3.55 \cdot 10^{-2}$**
U-238	**$-2.04 \cdot 10^{-2}$**	**$-1.87 \cdot 10^{-1}$**	**$3.00 \cdot 10^{-1}$**	$1.03 \cdot 10^{-3}$	**$4.30 \cdot 10^{-1}$**	**$-1.01 \cdot 10^{-1}$**
Pu-239	$-4.64 \cdot 10^{-3}$	**$-1.37 \cdot 10^{-2}$**	**$-3.10 \cdot 10^{-1}$**	**$1.48 \cdot 10^{-2}$**	**$3.56 \cdot 10^{-1}$**	**$-7.19 \cdot 10^{-1}$**
Pu-240	$-4.87 \cdot 10^{-4}$	$-1.39 \cdot 10^{-3}$	$-7.84 \cdot 10^{-3}$	$1.36 \cdot 10^{-3}$	**$1.25 \cdot 10^{-2}$**	$-2.11 \cdot 10^{-2}$
Pu-241	$-3.23 \cdot 10^{-5}$	$-1.26 \cdot 10^{-4}$	$2.66 \cdot 10^{-3}$	$1.11 \cdot 10^{-4}$	**$9.92 \cdot 10^{-3}$**	$-7.87 \cdot 10^{-3}$

SNEAK-7A: Uncertainty in β-eff (JENDL-4)

TOTAL=3 %

	Uncertainty (%)								
MAT.	**elastic**	**inelast.**	**(n,f)**	**(n,γ)**	**ν_{del}**	**ν_{prmt}**	**χ_p**	**χ_d**	**TOTAL**
U-235	0.002	0.022	0.033	0.002	**0.204**	0.007	**0.057**	0.028	0.22
U-238	**0.062**	**2.021**	**0.178**	**0.062**	**1.449**	**0.061**	**0.212**	**0.176**	**2.51**
Pu-239	0.015	**0.137**	**0.166**	**0.062**	**1.380**	**0.122**	**0.609**	**0.068**	**1.53**
Pu-240	0.002	0.011	0.005	0.006	**0.061**	0.004	0.018	0.002	0.06
Pu-241	0.001	0.002	0.004	0.001	0.049	0.004	0.008	0.002	0.05

JEZEBEL: Total uncertainty in β-eff: 2.2 %

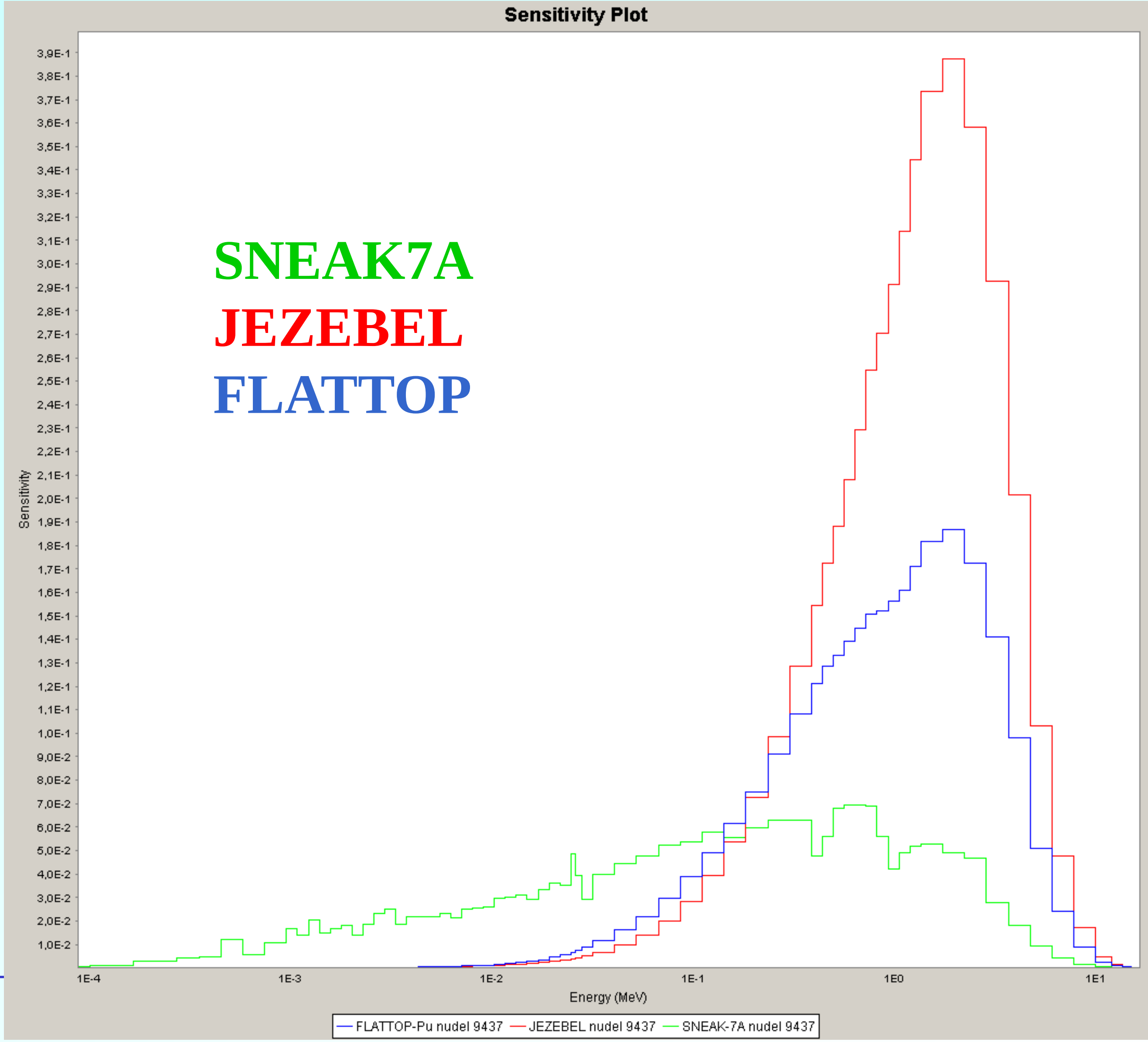
MAT	Sensitivity (%/%)					
	elastic	inelastic	(n,f)	(n,γ)	ν_{del}	ν_{pmt}
Pu-239	**$1.61 \cdot 10^{-2}$**	**$1.44 \cdot 10^{-2}$**	**$-3.82 \cdot 10^{-2}$**	$-2.57 \cdot 10^{-3}$	**$8.71 \cdot 10^{-1}$**	**$-8.85 \cdot 10^{-1}$**
Pu-240	$9.31 \cdot 10^{-4}$	$7.25 \cdot 10^{-4}$	$9.09 \cdot 10^{-3}$	$-1.93 \cdot 10^{-4}$	**$3.94 \cdot 10^{-2}$**	**$-3.00 \cdot 10^{-2}$**
Pu-241	$3.82 \cdot 10^{-5}$	$6.30 \cdot 10^{-5}$	$3.76 \cdot 10^{-3}$	$-1.35 \cdot 10^{-5}$	$6.34 \cdot 10^{-3}$	$-2.48 \cdot 10^{-3}$

MAT.	Uncertainty (%)								
	elastic	inelast.	(n,f)	(n,γ)	ν_{del}	ν_{pmt}	χ_p	χ_d	TOTAL
Pu-239	**0.068**	**0.098**	**0.065**	0.019	**2.130**	**0.210**	**0.518**	0.008	**2.21**
Pu-240	0.005	0.004	0.007	0.001	**0.188**	0.005	0.013	0.001	**0.19**
Pu-241	0.001	0.001	0.003		**0.031**	0.001	0.001		**0.03**

FLATTOP-Pu: Total uncertainty in β-eff: 2.6 %

FLATTOP	Uncertainty (%)								
MAT.	**elastic**	**inelast.**	**(n,f)**	**(n,γ)**	ν_{del}	ν_{pmt}	χ_p	χ_d	**TOTAL**
U-235	0.001	0.011	0.013	0.002	**0.061**	0.003	0.031	0.007	0.07
U-238	**0.165**	**1.890**	**0.171**	**0.090**	**1.105**	0.018	**0.171**	0.047	**2.21**
Pu-239	**0.078**	**0.428**	**0.233**	0.016	**1.242**	**0.202**	**0.187**	**0.092**	**1.37**
Pu-240	0.006	0.018	0.007	0.001	**0.106**	0.003	0.009	0.003	0.11
Pu-241	0.001	0.003	0.001		0.020	0.001	0.001	0.001	0.02

Sensitivity of β-eff to delayed nu-bar of Pu-239



Objectives / Conclusions (β_{eff} sensitivities)

- New and relatively simple method was proposed to evaluate the uncertainty in beta-effective due to the basic nuclear data. It is obtained by deriving the Bretscher's (prompt k-ratio) β_{eff} expression with respect to the basic nuclear data.
- The method was implemented in SUSD3D and applied to the fast benchmarks SNEAK-7A, JEZEBEL and FLATTOP. seem the method provides reasonable results, in rough agreement with other methods used in the past.
- According to the JENDL-4 covariance data the uncertainty in β_{eff} is mainly due to the uncertainties in delayed (and prompt) neutron yields, inelastic and elastic scattering and fission cross sections, as well as the prompt and delayed fission spectra. The total uncertainty in β_{eff} was found to be between 2.2 to 3 % for the studied cases.